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International Journal of Heat and Mass Transfer 44 (2001) 207–214

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Turbulent film boiling on a horizontal cylinder

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Received 3 February 1997; received in revised form 18 February 1999

Abstract

The turbulent film boiling on a horizontal cylinder is formulated and solved giving due importance to thermal radiation. In the analysis the test surface is maintained under isothermal conditions and the thermophysical property variation of the vapor due to the steep temperature gradients in the vapor boundary layer is included in the formulation. The analysis is checked with some experimental data relevant to turbulent film boiling. The agreement between the two is found to be very satisfactory.

1. Introduction

Film boiling studies have been extensively reviewed by many authors, such as Jordan [1] and Kalinin et al. [2]. Many of the studies are for the case of laminar vapor flow adjacent ot the solid boundary of the test surface. The boundary layer analysis of Sakurai et al. [3,4] is for the smooth wave free laminar regime of the vapor flow. The turbulent film boiling studies are very limited in comparison to those devoted to laminar film boiling. Some of the important contributions in turbulent film boiling are due to Bankoff [5], Coury and Dukler [6], Borishansky [7], Westwater and Hsu [8] and Sarma et al. [9,10].

The present investigation is aimed at undertaking the study of some of the aspects that still remain unexplored. Film boiling invariably occurs at very high wall temperatures. Consequently, the physical properties of the vapors are invariably influenced by the temperature gradients and hence the physical property variation with respect to temperature should be included in the analysis. In a previous paper, Sarma et al. [10] considered the case of constant heat flux. However, the case of constant wall temperature is yet to be solved. Further, at high temperatures of the wall, the influence of thermal radiation is a matter of importance since it can play a significant role in altering velocity and temperature profiles across the vapor film. Hence, the present paper is intended to determine the combined influence of thermal radiation and physical property variation with respect to temperature under isothermal conditions of the wall of the horizontal cylinder for the turbulent flow regime of the vapor. Further, the analysis is checked for validity with relevant experimental data.

2. Physical model and formulation

The physical model and the coordinate system are shown in Fig. 1. The test surface is a horizontal cylinder and the wall temperature is at a magnitude to ensure stable film boiling on the cylinder immersed in

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0017-9310/01/$ - see front matter
PII: S0017-9310(00)00061-2

Nomenclature

C_p	specific heat at constant pressure, kJ/kg/K
g	acceleration due to gravity, m/s^2
Gr_D	modified Grashof number, $g(\rho_l - \rho_s)D^3/(\rho_s \nu_s^2)$
h	average heat transfer coefficient, W/m^2/K
h_{fg}	latent heat of vaporisation, kJ/kg
k	thermal conductivity, W/m/K
k^+	$k(T)/k(T_s)$
D	diameter of the tube, m
D^+	dimensionless diameter
Nu_D	local Nusselt number (hD/k_s)
$\overline{Nu}_D$	mean Nusselt number
Pr	Prandtl number ($\mu C_p/k_s$),
R	radius of the tube, m
S	heat capacity parameter ($C_p T_s/h_{fg}$)
T	temperature K
T^+	dimensionless temperature $(T - T_s)/(T_w - T_s)$
u	velocity in x-direction, m/s
u^*	shear velocity, $(\tau/\rho)^{0.5}$
u^+	dimensionless velocity (u/u^*)
v	velocity normal to the direction of flow, m/s
y	distance normal to the direction of flow, m
y^+	dimensionless distance normal to the direction of flow (yu^*/ν_s)

Greek symbols

$\Delta\delta^+$	increment in dimensionless vapor boundary layer thickness
δ	vapor boundary layer thickness, m
δ^+	dimensionless vapor boundary layer thickness ($\delta u^*/\nu_s$)
ε	emissivity
ε_m	eddy diffusivity, m^2/s
ϕ	temperature ratio (T_w/T_s)
σ	Stefan–Boltzmann constant, W/m^2 K^4
ρ	density, kg/m^3
μ	dynamic viscosity, kg/ms
μ^+	$\mu(T)/\mu(T_s)$
ν	kinematic viscosity, m^2/s
τ	shear stress, N/m^2
θ	angle measured from lower stagnation point

Subscript

i	interface
l	liquid
s	vapor at saturation temperature
θ	local
v	vapor
w	wall

an ambient liquid at the saturation temperature corresponding to the given system pressure. It is thought that the vapor bubbles leaving at the upper stagnation point of the cylinder agitate the liquid bulk to a degree enough to induce turbulent regime of vapor flow adjacent to the cylinder. Generally, for free convective film boiling the demarcation between the flow regimes is characterised by the Grashof number. In the present study, the value of Grashof number is limited to 10^9 and more. Further, for flow around cylinders, boundary layer separation can occur at a certain angle measured from the lower stagnation point. However, in the present analysis, boundary layer separation is excluded on the assumption that all vapor generated around the periphery accumulates and gets released in the form of vapor slugs at regular intervals at the upper stagnation point. In the analysis, the effect of thermo-physical property variation is included together with the thermal radiation effects on the process of vaporisation.

The conservation of mass, momentum and thermal energy of the vapor film for the configuration and coordinate system shown in Fig. 1 are as follows:

Continuity

$$\frac{\partial}{\partial x}(\rho_v u) + \frac{\partial}{\partial y}(\rho_v v) = 0 \tag{1}$$

Momentum

$$\rho_v \left\{ u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right\} = g(\rho_i - \rho_v)\sin\theta + \frac{\partial}{\partial y}\left\{ \mu\left(1 + \frac{\varepsilon_m}{\nu}\right)\frac{\partial u}{\partial y} \right\} \tag{2}$$

Energy

$$\rho_v C_p \left\{ u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right\} = \frac{\partial}{\partial y}\left\{ k\left(1 + \frac{\varepsilon_H}{\alpha}Pr\right)\frac{\partial T}{\partial y} \right\} \tag{3}$$

The boundary conditions are as follows

$u = v = 0; \quad y = 0; \qquad 0 \le \theta \le \pi$

$$\tau = \tau_{iv}; \quad y = \delta; \qquad 0 \le \theta \le \pi$$

$$T = T_w; \quad y = 0; \qquad 0 < \theta < \pi$$

$$T = T_s; \quad y = \delta; \qquad 0 \le \theta \le \pi \tag{4}$$

At the interface the phase transformation is given by the condition

$$\frac{d}{d\theta}\int_0^{\delta} \rho_v u \, dy = \frac{-k_w R}{h_{fg}} \frac{\partial T}{\partial y}\bigg|_{y^+=0} + \varepsilon\sigma R\{T_w^4 - T_s^4\} \tag{5}$$

Eq. (5) assumes that the total heat of vaporisation in the vapor film is equal to the heat flux at the wall due to conduction across the film, and the component of flux corresponding to thermal radiation.

The following assumptions are made to render the array of Eqs. (1)–(5) into tractable form.

- The eddy thermal diffusivity ε_H is taken same as eddy kinematic viscosity ε_m and the turbulent Prandtl number is unity.
- The surface of the cylinder is under isothermal conditions at a specified temperature, T_w.
- Properties such as (k, μ) are temperature-dependent and can be represented in an appropriate functional form for the given system.

For example, the property variations for vapors of liquid nitrogen are given by the relationship as follows:

$$\frac{\mu}{\mu_s} = \mu^+ = 2.656 - 3.804\left(\frac{T}{T_s}\right) + 2.148\left(\frac{T}{T_s}\right)^2 \tag{6}$$

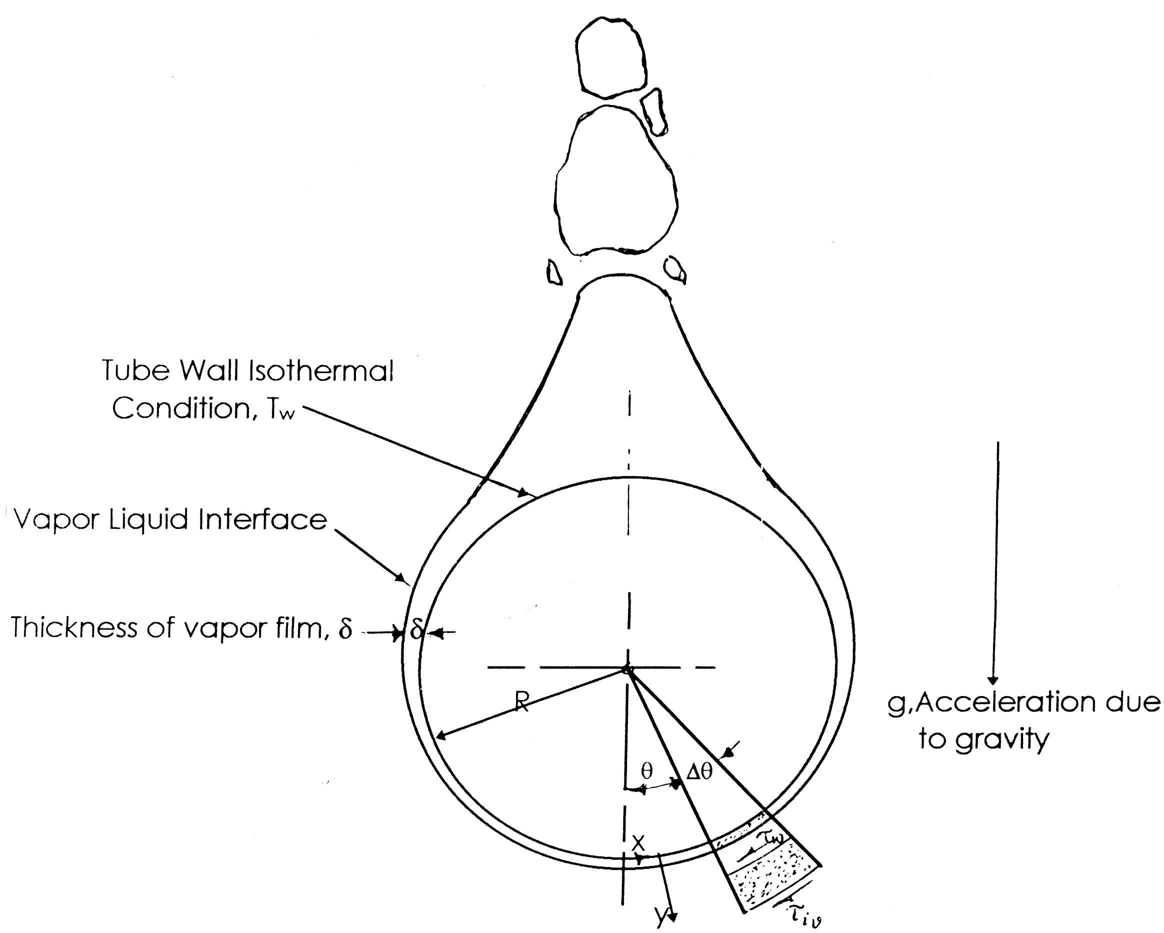

Fig. 1. Configuration and coordinate system.

$$\frac{k}{k_s} = k^+ = 236.1 - 811.5\left(\frac{T}{T_s}\right) + 1045.4\left(\frac{T}{T_s}\right)^2 - 596.2\left(\frac{T}{T_s}\right)^3 + 127.2\left(\frac{T}{T_s}\right)^4 \quad (7)$$

where (μ_s, k_s) are respectively the absolute viscosity and thermal conductivity of nitrogen at T_s, the saturation temperature corresponding to a system pressure of 1 atm.

- The eddy diffusivity of momentum ε_m is given by Kato's expression [11] i.e.,

$$\frac{\varepsilon_m}{\nu_s} = 0.4y^+\left[1 - e^{-ay^{+2}}\right] \quad (8)$$

where

$$a = 0.0017; \quad y^+ = \frac{yu^*}{\nu_s}$$

- The component of thermal radiation from the test surface can be assessed considering the vapor to be a transparent medium relative to thermal radiation.
- In Eq. (2) the inertial force terms are excluded with the assumption that the vapor film dynamics is determined by the balance between the buoyant force and the turbulent shear in the vapor film.
- The interfacial shear stress 'τ_{iv}' at the vapor–liquid interface is of the same order as that which would prevail at the solid boundary i.e., $\tau_{iv} = -\tau_w$ at $y = \delta$. This assumption is tantamount to rigid boundary condition for laminar flow condition of the vapor. However, for turbulent flow conditions, the velocity of the vapor is finite due to the presence of the eddy diffusivity terms (ε_m/ν) in the equation of motion. In the literature related to turbulent film boiling the assumptions are either $\tau_{iv} > \tau_w$ or $\tau_{iv} < \tau_w$, respectively, as per Borishansky [7] and Bankoff [5]. Within the framework of these assumptions the problem is formulated as follows in dimensionless forms by simplifying Eq. (1)–(3)
- The pressure across the boundary layer is constant and the density variation across the boundary layer is given by the relationship.

$$\rho T = \rho_w T_w = \rho_s T_s$$

3. Velocity profile

Thus, the momentum balance equation can be simplified as follows making use of the further assumption that the motion of vapor is governed by the balance between the eddy viscous forces and the buoyant forces as follows:

$$\frac{du^+}{dy^+} = \frac{\left[1 - 2y^+/\delta^+\right]}{\phi\mu^+\left[1 + \frac{\varepsilon_m}{\nu_s}\frac{1}{\mu^+\left[1 + T^+(\phi - 1)\right]}\right]} \quad (9)$$

The boundary condition at the wall is

$$u^+ = 0 \quad \text{at } y^+ = 0$$

4. Temperature profile

Similarly, the convection equation can be approximated as follows:

$$\frac{dT^+}{dy^+} = \frac{1}{k^+}\left\{\frac{\frac{dT^+}{dy^+_{|y^+=0}}}{1 + \frac{\varepsilon_m}{\nu_s}\frac{Pr}{\mu^+\left[(1 + T^+)(\phi - 1)\right]}}\right\} \quad (10)$$

The boundary conditions at the wall and the interface are as follows:

$$\text{at } y^+ = 0; \quad T^+ = 1$$

$$y^+ = \delta^+; \quad T^+ = 0$$

Evidently, Eqs. (9) and (10) are to be solved simultaneously, since the temperature dependent functions appear in these equations. Further, it can be observed from these equations that for very low values of δ^+, the term $\varepsilon_m/\nu \to 0$, and the solutions tend to the particular case of laminar film boiling with temperature dependent property variation accounted for in the analysis. This is tantamount to the fact that the analysis takes care of both laminar and turbulent regimes that can occur on the periphery of the cylinder having high magnitudes of Grashof numbers.

The phase transformation at the vapor–liquid interface is given by the following mass balance:

$$\frac{d}{d\theta}\int_0^{\delta^+}\frac{u^+ dy^+}{\left[1 + T^+(\phi - 1)\right]} = -A\frac{D^+}{2}(\phi - 1)k^+\frac{\partial T^+}{\partial y^+}\bigg|_{y^+=0} + N_R\left[\phi^4 - 1\right]A \quad (11)$$

where

$$A = \left[\frac{C_p T_s}{h_{fg}}\frac{1}{Pr}\right]; \quad T^+ = \left[\frac{T - T_s}{T_w - T_s}\right];$$

$$N_R = \left[\frac{\varepsilon\sigma T_s^3 R}{k_s}\right]; \quad \phi = \frac{T_w}{T_s}$$

$$D^{+} = \left[0.5\delta^{+} Gr_D \phi \sin\theta\right]^{1/3} \tag{12}$$

The local heat transfer coefficients are evaluated from the relationship

$$Nu_D = -k^{+}(T^{+}-1)\left\{0.5\, Gr_D \delta^{+}\phi \sin\theta\right\}^{1/3} + 2\, N_R\left[\frac{\phi^4-1}{\phi-1}\right] \tag{13}$$

Further, the average heat transfer coefficients can be evaluated from the equation

$$Nu = \frac{1}{\pi}\int_0^{\pi} Nu_D\, \mathrm{d}\theta \tag{14}$$

5. Heat transfer at $\theta = 0$

The heat transfer conditions at the lower stagnation point are predominantly identical to the Bromley's analysis for laminar film boiling. However, when the liquid film thickness δ^{+} crosses a value equal to 10, it is assumed that the flow transits to turbulent flow regime. The formulation has the advantage that such a situation is automatically taken care of by the equations.

The vapor film thickness at the lower stagnation point, i.e., $\theta = 0$ can be obtained following the Nusselt type of analysis for condensation as follows:

$$\frac{\delta}{D}\Big|_{\theta=0} = 1.56\left\{\frac{k_{\mathrm{v}}\Delta T \mu_{\mathrm{v}}}{g(\rho_{\mathrm{l}}-\rho_{\mathrm{v}})\rho_{\mathrm{v}} D^3 h_{\mathrm{fg}}}\right\}^{1/4} \tag{15}$$

It follows at $\theta = 0$, the Nusselt number is given by the following expression.

$$Nu|_{\theta=0} = 0.63\left[\frac{gD^3}{\nu_{\mathrm{v}}^2}\frac{\rho_{\mathrm{l}}-\rho_{\mathrm{v}}}{\rho_{\mathrm{v}}}\frac{h_{\mathrm{fg}}}{C_{\mathrm{p}}\Delta T}Pr\right]^{1/4} \tag{16}$$

For laminar film boiling the shear velocity i.e., $\sqrt{\tau_{\mathrm{w}}/\rho_{\mathrm{v}}}$ is given by the expression

$$u^{*} = \left[0.5g\left(\frac{\rho_{\mathrm{l}}-\rho_{\mathrm{v}}}{\rho_{\mathrm{v}}}\right)\delta \sin\theta\right]^{1/2} \tag{17}$$

Hence, $\delta^{+} = 0$ at $\theta = 0$ though the vapor film thickness δ is finite.

6. Method of obtaining the solution

The problem is formulated mathematically through Eqs. (6)–(14) and the method of solving these equations is outlined for the range $0 < \theta < \pi$

1. At $J = 1$ i.e., $\delta^{+} = 0$ the input parameters corresponding to the system conditions are assumed, i.e., Gr, $(C_{\mathrm{p}}(T_{\mathrm{s}}/h_{\mathrm{fg}})$, N_R.
   Though δ^{+} is zero, the Nusselt number is given by the Eq. (16).
2. By advancing to node $J = J + 1$ and presuming that δ^{+} increases by a value of $\Delta\delta^{+} = 0.5$, the velocity and temperature profiles are obtained with the aid of Eqs. (9) and (10) for the system parameters assigned.
3. With the aid of Eqs. (11) and (12), the location θ is computed by a trial and error procedure for the assigned value of δ^{+} at this node by writing down the Eq. (11) in finite difference form. Further, the local heat transfer coefficient is estimated.
4. δ^{+} is increased further by $\Delta\delta^{+}$ for the node $J = J + 1$; steps (2) and (3) are repeated. The calculations are thus carried out covering the range $0 < \theta < \pi$. From the data of local heat transfer coefficients, the average heat transfer coefficients are calculated from Eq. (14).

7. Results and discussion

In Fig. 2 the variation of vapor film thickness around the cylinder from the lower stagnation point to the upper stagnation point is shown plotted. It can be seen that the variation of the dimensionless vapor film thickness δ^{+} is monotonic and the increase in the vapor film thickness is due to the contribution of vapour generation at the interface to the boundary

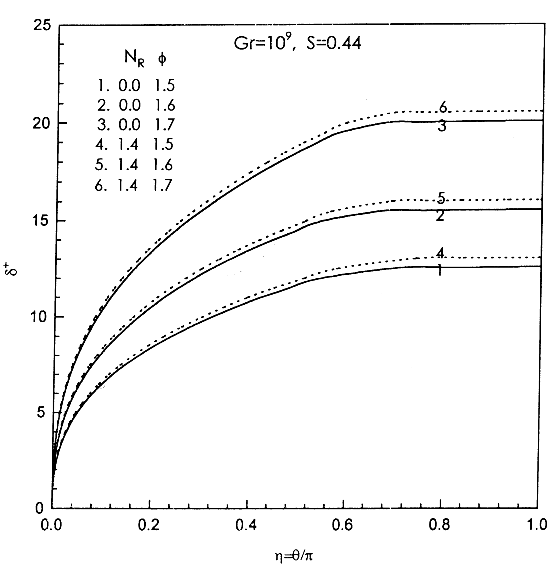

Fig. 2. Variation of film thickness.

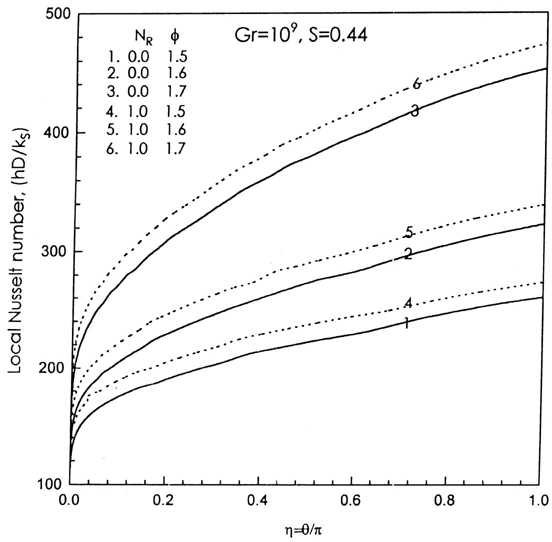

Fig. 3. Variation of local heat transfer coefficient.

layer. Further, when thermal radiation is included in the analysis the vapour film becomes thicker, i.e., larger than the values obtained with the radiation term excluded. In Fig. 3 it can be seen that the local values of turbulent film boiling heat transfer coefficient are found to increase with the wall temperature. The total heat transfer coefficients with the thermal radiation term included in the analysis are found to be higher in magnitude than the values for the no radiation case.

The effect of property variation with temperature can be seen to reflect on the velocity gradients at the solid boundary. For example, the variation of velocity gradients at the wall decrease with the increase in wall temperature as can be seen in Fig. 4. An increase in the temperature leads to an increase in the viscosity of the vapors and hence the shear resistance at the wall increases. The dimensionless shear resistance at the wall can be represented by the polynomial

$$\left.\frac{du^+}{dy^+}\right|_{y^+=0} = 3.691 - 3.604\phi + 0.918\phi^2 \tag{18}$$

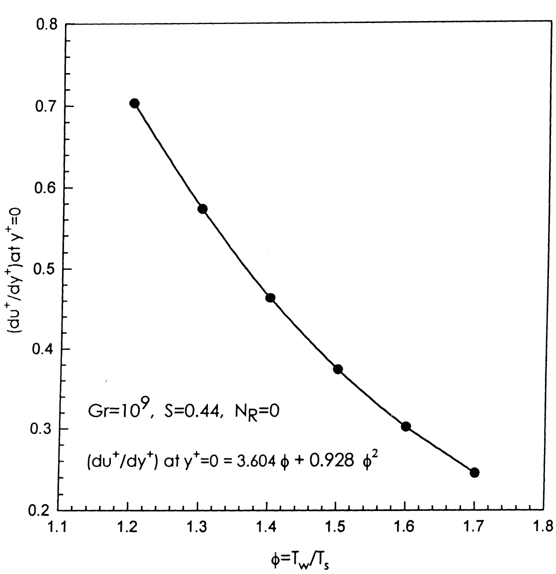

Fig. 4. Variation of dimensionless shear at wall.

It is evident from Eq. (18) for adiabatic conditions, when ϕ tends to unity, $du^+/dy^+ = 1$. Such a situation arises only when physical properties are considered to be temperature independent. In reality, ϕ must have a value greater than ϕ_c, critical value that would lead to the onset of stable film boiling. Eq. (18) would merely serve as a check to establish the validity of the numerical procedure employed in the investigation. For isothermal conditions without heat addition the velocity gradient at the wall tends to unity for turbulent boundary layer. The validation of the present analysis is shown in Fig. 5 with the theoritical data shown as thick lines together with the experimental data for Freon-12 and liquid nitrogen. The coordinate system chosen is identical to the one employed by Frederking and Clark [12] except that $a/g = 1$. It can be inferred that the theoretical formulation favourably responds to the practical situation of turbulent film boiling heat transfer for the ranges of parameters studied in the article. The present analysis is consistent with observations made by Hsu and Westwater [8] viz., increase in ΔT enhances the average film boiling heat transfer coefficient, i.e., the vapor film Reynolds number at $\theta \to \pi$ is more than the critical value demarcating the transition from laminar to turbulent flow.

A dimensionless equation is proposed by applying regression to the computed values and the turbulent film boiling heat transfer coefficients for isothermal condition of the cylinder for the following ranges of parameters is as follows:

$1.4 < \phi < 1.8$

$0.2 < C_p T_s / h_{fg} < 0.8$

$10^9 < Gr < 10^{12}$

$N_R = 0$

$$Nu = 0.272\, Ra^{1/3} (\phi - 1)^{1.25} \tag{19}$$

Where Ra is modified Rayleigh number as shown on the abscissa of Fig. 5, i.e., $Ra = Gr_D Pr[0.5 + 1/$

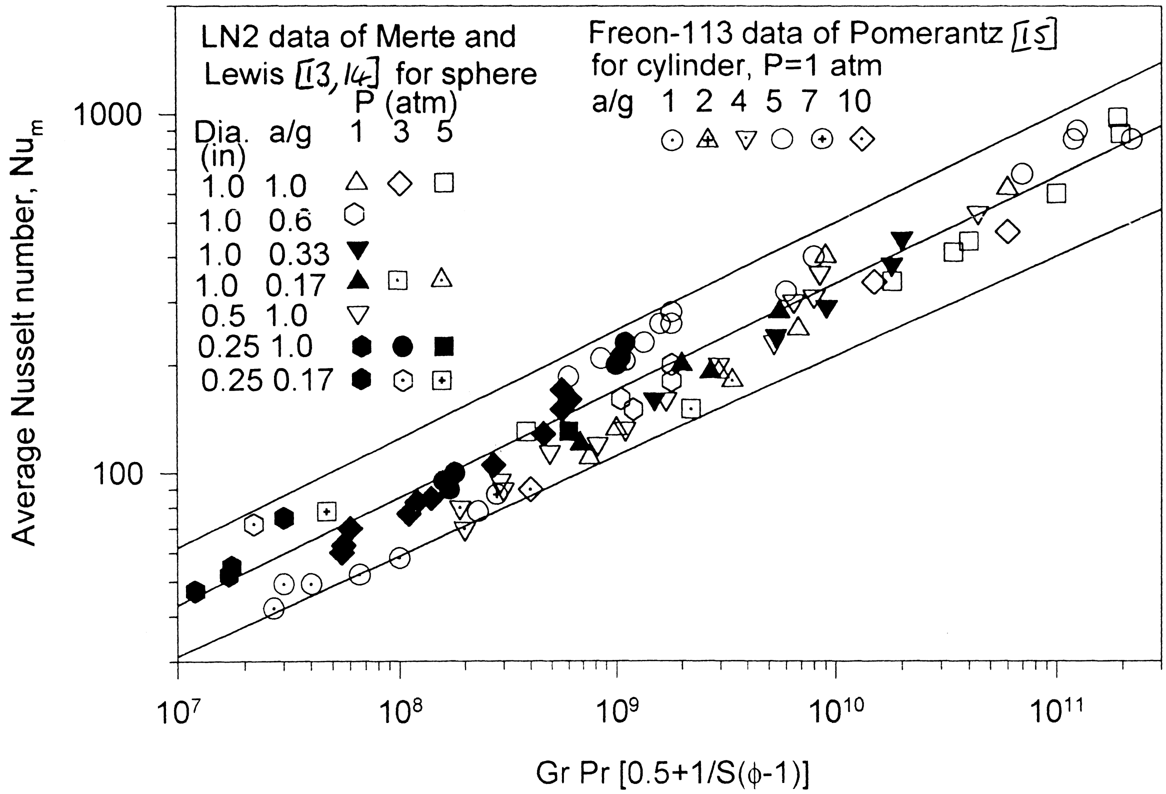

Fig. 5. Comparison of the present theory with experimental data in literature.

$S(\phi - 1)$]. The regression analysis indicated an average deviation of 4.9% and a standard deviation of 5.8%. The regression is accomplished in terms of the criteria defined by Frederking and Clark [12]. However, Eq. (19) contains an additional term ϕ, which accounts for physical property variation as a function of wall temperature. When $\phi \to 1$, the Nusselt number tends to zero. Equation holds good for the case of stable film boiling i.e., $\phi \geq \phi_c$, where ϕ_c is the critical value.

8. Conclusions

The present study, deals with turbulent film boiling on a horizontal cylinder under isothermal conditions of the test surface. It is observed that the present equation is similar to the dimensionless equation proposed by Frederking and Clark [12] except that an additional temperature ratio term appears. The assumptions employed in the formulation are found to be rational; at least for the ranges of parameters investigated. The average heat transfer coefficient can be evaluated from Eq. (19).

Acknowledgements

This is to acknowledge that the material presented herein was accomplished during the stay of the first author in the Heat Transfer Laboratory at Rensselaer Polytechnic Institute, Troy, New York as a Visiting Researcher.

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